34.12 A combined cycle is composed of a gas turbine cycle with heat recovery from the exhaust gases using a heat recovery steam generator. The gas turbine cycle without heat recovery is 25% efficient, has a net work output of $100\frac{Btu}{lb_m}$, and a heat transfer per unit mass of $400\frac{Btu}{lb_m}$. The heat recovery portion of the cycle can be modeled as a 35% efficient Rankine cycle and has a steam mass flow rate that is one tenth of the gas turbine mass flow rate. What is the thermal efficiency of the combined cycle?

A. 26%

B. 28%

C. 42%

D. 52%

Refer the the Combined Cycle diagram in the reference handbook. A combined cycle consists of a gas turbine cycle with a heat recovery steam generator making use of the waste heat to obtain additional work output from a second, smaller turbine. As a starting point, recall that the efficiency of a gas turbine cycle is the ratio of net work to heat in, which is consistent with the information given, prior to the addition of the heat recovery portion. Notice both the work and heat in this problem are given on a per unit mass basis.

$$\eta_{no\,recovery} = \frac{\dot{w}_{out}}{\dot{q}_{in}} = \frac{100\frac{Btu}{lb_m}}{400\frac{Btu}{lb_m}} = 0.25$$

Since one fourth of the input heat is converted to output work, the remainder leaves the cycle as waste heat. The quantity of waste heat becomes the heat input to the heat recovery steam generator.

$$\dot{q}_{in,HRSG} = 400\frac{Btu}{lb_m} - 100\frac{Btu}{lb_m} = 300\frac{Btu}{lb_m}$$

The heat recovery cycle can be modeled as a Rankine Cycle with 35% efficiency. The efficiency of a Rankine cycle is the ratio of net work to input heat, where net work is the difference between the turbine output work and the pump input work. It is reasonable to neglect the pump input work as it is often miniscule. Determine the net work for the heat recovery cycle.

$$\eta_{Rankine} = \frac{\dot{w}_{net}}{\dot{q}_{in}}$$

$$\dot{w}_{net,Rankine} = \eta\dot{q}_{in} = (0.35)\left(300\frac{Btu}{lb_m}\right) = 105\frac{Btu}{lb_m}$$

Calculate the combined cycle net work by adding the work output from the gas turbine cycle without recovery to the work output from the recovery cycle. Since the mass flow rate of the recovery cycle is one tenth of the gas turbine cycle, its contibution to the overall work output must be scaled accordingly.

$$\dot{w}_{out,combined} = 100\frac{Btu}{lb_m} + (0.1)\left(105\frac{Btu}{lb_m}\right) = 110.5\frac{Btu}{lb_m}$$

Calculate the efficiency of the combined cycle. The denominator remains unchanged from its original value because the heat input to the recovery cycle was previously considered waste heat. The waste heat should not be added to $\dot{q}_{in}$ as that would be double counting.

$$\eta_{combined} = \frac{\dot{w}_{out}}{\dot{q}_{in}} = \frac{110.5\frac{Btu}{lb_m}}{400\frac{Btu}{lb_m}} = 0.276 \approx 28\%$$

Answer B